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Chaotic modulation of the solar cycle†

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The Sun's magnetic activity varies cyclically, with a well-defined mean period of about 11 years. At the beginning of a new cycle, spots appear at latitudes around $\pm 30°$; then the zones of activity expand and drift towards the equator, where they die away as the new cycle starts again at higher latitudes. Active regions are typically oriented parallel to the equator, with oppositely directed magnetic fields in leading and following regions. The sense of these fields is opposite in the two hemispheres and reverses at sunspot minimum. So the magnetic cycle has a 22-year period, with waves of activity that drift towards the equator. Sunspot records show that there was a dearth of spots in the late 17th century – the Maunder minimum – which can also be detected in proxy records. Galactic cosmic rays, whose incidence is modulated by magnetic fields in the solar wind, form radioactive isotopes in the Earth's atmosphere. Anomalies in the abundances of ^{14}C (in trees) and ^{10}Be (in polar ice cores) show that grand minima of solar activity have occurred irregularly over the past 8000 years, with a characteristic timescale of 200 years. The sample of magnetic variations can be enlarged by studying other lower-main sequence stars, which also show the effect of varying parameters such as the angular velocity Ω (Weiss 1994). Magnetic activity depends on Ω, and stars spin down owing to magnetic braking. Slow rotators exhibit cycles like the Sun's and about 30% of them are quiescent, as if undergoing grand minima.

Solar activity is aperiodic but the sunspot record, extending over less than 300 years, is too short to distinguish between stochastic effects and deterministic chaos (Weiss 1990). However, it is reasonable to assume that we are dealing with a chaotically modulated oscillator subject to weak stochastic perturbations. It is generally accepted that solar and stellar magnetic fields are produced by dynamo action, and that the growth of the field is limited by effects of the nonlinear Lorentz force. Both self-consistent dynamo models (where the relevant partial differential equations are solved numerically, after adopting laminar diffusivities) and mean-field ($\alpha\omega$) dynamos provide examples of aperiodic oscillations. Moreover, the behaviour of mean-field dynamos depends on the dynamo number, $D \propto \Omega^2$. Parker (1979) proposed that there are two different convective states in a rotating star, only one of which is unstable to magnetic perturbations, and that the system moves from this state towards the other, which is hydrodynamically unstable, so giving rise to modulation. Thus we expect to find an initial Hopf bifurcation, leading to periodic magnetic cycles, followed by a secondary Hopf bifurcation that gives rise to periodic modulation and then by a transition to chaos.

Generic patterns of behaviour can be studied in low-order (toy) models, which

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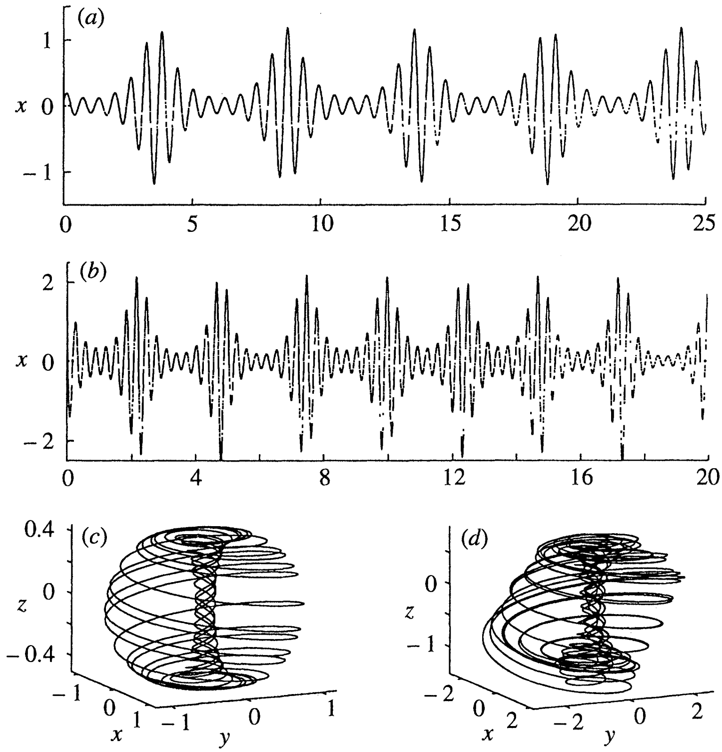

Figure 1. (*a*) The poloidal field x as a function of time t for a quasi-periodic (periodically modulated) solution. (*b*) The same, but for a chaotically modulated solution. (*c*) Perspective view of the two-torus, corresponding to case *a*. (*d*) The chaotic attractor for case *b*.

help to characterize the behaviour of the real dynamo (Weiss *et al.* 1984). A third-order system is necessary to produce a two-torus and is sufficient for chaos. Tobias *et al.* (1994) have recently constructed a minimal model that reproduces the essential features of solar activity. In this model all the hydrodynamics is collapsed onto the z-axis of cylindrical polar coordinates, with two fixed points generated in a saddle-node bifurcation. The poloidal and toroidal components of the magnetic field are represented by $x + \mathrm{i}y = r\mathrm{e}^{\mathrm{i}\phi}$. Then the appropriate normal form equations for a saddle-node/Hopf bifurcation are

$$\begin{aligned}\dot{z} &= \mu - z^2 - r^2 - bz^3,\\ \dot{r} &= \lambda r + arz + cr^2 z\cos\phi,\\ \dot{\phi} &= \omega - crz\sin\phi,\end{aligned}$$

where λ, μ are parameters that depend on D and hence on Ω, while a, b, c and ω are constants (Kirk 1991). This system already exhibits the required behaviour. The primary Hopf bifurcation leads to periodic magnetic cycles, with trajectories attracted to a limit cycle in phase space. After the second Hopf bifurcation, solutions are quasi-periodic, as illustrated in figure 1 *a*, with trajectories that lie on the two-torus shown in figure 1 *c*. This corresponds to periodically modulated activity. The presence of a heteroclinic tangle, with horseshoes and associated chaos (Guckenheimer & Holmes 1986), leads to aperiodically modulated solutions,

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like that in figure 1 *b*; the corresponding trajectories lie on the chaotic attractor displayed in figure 1 *d*. Furthermore, the form of these chaotic solutions depends on the values chosen for the parameters and the constants.

This simple model system therefore succeeds in capturing the essential features of the solar cycle. Since these are normal-form equations, the behaviour is robust and likely to be found in higher order systems too. This approach demonstrates, however, that it is possible to obtain generic behaviour without needing to know details of the dynamo process. Indeed, given a supercritical Hopf bifurcation from an invariant hydrodynamic subspace, we could predict the existence of chaotic modulation. It will be interesting to see whether a similar approach can be used to explore the connection between solar magnetic activity and climatic variations.

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